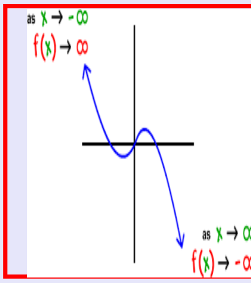
Math 245

Spring 2022

Lecture 37



Consider $P(x) = x^3 + 3x^2 - 13x - 15$

At most 3 Zeros (Roots, Solutions, x-Ints)

Polynomial, Descending order, real Coef., NonZero Const.

$P(x) = x^3 + 3x^2 - 13x - 15$

1 ⇒ 1 positive Zero

$P(-x) = (-x)^3 + 3(-x)^2 - 13(-x) - 15$

$= -x^3 + 3x^2 + 13x - 15$

1   2 ⇒ 2 or 0 Negative Zeros

Pos.	Neg.	Complex
1	2	0
1	0	2

} sum = deg

↑ even

List of Possible Rational Zeros ⇒ $\frac{\pm 1, \pm 3, \pm 5, \pm 15}{\pm 1}$ ⇒ ±1, ±3, ±5, ±15

Is 3 a Zero?

$$\begin{array}{r|rrrr} 3 & 1 & 3 & -13 & -15 \\ & & 3 & 18 & 15 \\ \hline & 1 & 6 & 5 & 0 \end{array}$$

Yes,

maybe is a repeated Zero
NO

$$\begin{array}{r|rrr} 3 & 1 & 6 & 5 \\ & & 3 & 27 \\ \hline & 1 & 9 & 32 \end{array}$$

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is -1 a zero? Yes

$$\begin{array}{r|rrr} -1 & 1 & 6 & 5 \\ & & -1 & -5 \\ \hline & 1 & 5 & 0 \end{array}$$

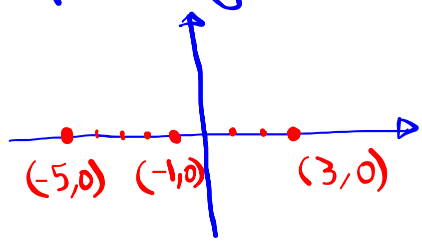
$P(x) = (x-3)(x+1)(x+5)$

$\to x+5=0$

$\boxed{x=-5}$

At most 3 Zeros:

$\boxed{-5, -1, 3} \Rightarrow 1$ pos. & 2 Neg. Zeros



$P(x) = 3x^4 - 7x^3 - 6x^2 + 12x + 8$

At most 4 Zeros

$P(x) = 3x^4 - 7x^3 - 6x^2 + 12x + 8$

1 2

$\boxed{\text{2 or 0 positive Zeros}}$

$P(-x) = 3x^4 + 7x^3 - 6x^2 - 12x + 8$

1 2

$\boxed{\text{2 or 0 Negative Zeros}}$

Pos.	Neg.	Complex
2	2	0
2	0	2
0	2	2
0	0	4

$P(x) = 3x^4 - 7x^3 - 6x^2 + 12x + 8$

List of all Possible rational Zeros $\Rightarrow \dfrac{\pm 1, \pm 2, \pm 4, \pm 8}{\pm 1, \pm 3}$

$\pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{8}{3}$

Is 4 an upper bound?

$$\begin{array}{r|rrrrr} 4 & 3 & -7 & -6 & 12 & 8 \\ & & 12 & 20 & 56 & 272 \\ \hline & 3 & 5 & 14 & 68 & 280 \end{array}$$

$\underbrace{3 \quad 5 \quad 14 \quad 68 \quad 280}$ ← Nonnegative

4 is an upper bound

Is -2 a lower bound?

$$\begin{array}{r|rrrrr} -2 & 3 & -7 & -6 & 12 & 8 \\ & & -6 & 26 & -40 & 56 \\ \hline & 3 & -13 & 20 & -28 & 64 \end{array}$$

Alternating Signs

-2 is a lower bound.

$P(x) = 3x^4 - 7x^3 - 6x^2 + 12x + 8$

Is 2 a Zero?

$$\begin{array}{r|rrrrr} 2 & 3 & -7 & -6 & 12 & 8 \\ & & 6 & -2 & -16 & -8 \\ \hline & 3 & -1 & -8 & -4 & 0 \end{array}$$

Remainder $= 0$

2 is a Zero.

Is 2 a repeated Zero?

$$\begin{array}{r|rrrr} 2 & 3 & -1 & -8 & -4 \\ & & 6 & 10 & 4 \\ \hline & 3 & 5 & 2 & 0 \end{array}$$

Rem. $= 0$

2 is a Zero again.

$P(x) = (x-2)(x-2)(3x^2 + 5x + 2)$

Solve $3x^2 + 5x + 2 = 0$ by quadratic formula

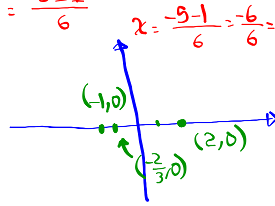
$a = 3$
$b = 5$
$c = 2$

$b^2 - 4ac = 5^2 - 4(3)(2) = 25 - 24 = 1$

$x = \dfrac{-5 \pm \sqrt{1}}{2(3)} = \dfrac{-5 \pm 1}{6}$

$x = \dfrac{-5+1}{6} = \dfrac{-4}{6} = \boxed{\dfrac{-2}{3}}$

$x = \dfrac{-5-1}{6} = \dfrac{-6}{6} = \boxed{-1}$



$P(x) = x^4 + 13x^2 + 36$

At most 4 Zeros

$P(x) = x^4 + 13x^2 + 36$

No Variations $\Rightarrow$ 0 pos. Zeros.

$P(-x) = x^4 + 13x^2 + 36$

No variations $\Rightarrow$ 0 Neg. Zeros.

List of all possible rational Zeros: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18, \pm 36$

Pos.	Neg.	Complex
0	0	4

$P(x) = x^4 + 13x^2 + 36$

$= (x^2 + 4)(x^2 + 9)$

$x^2 + 4 = 0$ $\qquad$ $x^2 + 9 = 0$

$x^2 = -4$ $\qquad$ $x^2 = -9$

$\boxed{x = \pm 2i}$ $\qquad$ $\boxed{x = \pm 3i}$

$P(x) = x^3 + 1$

At most 3 Zeros

$P(x) = x^3 + 1$ $\Rightarrow$ No Variations $\Rightarrow$ 0 pos. Zeros.

$P(-x) = -x^3 + 1$ $\Rightarrow$ 1 variation $\Rightarrow$ 1 neg. Zeros

Pos.	Neg.	Complex
0	1	2

List of all possible Rational Zeros: ± 1

Is -1 a Zero?

$$\begin{array}{r|rrrr} -1 & 1 & 0 & 0 & 1 \\ & & -1 & 1 & -1 \\ \hline & 1 & -1 & 1 & 0 \end{array}$$

$P(x) = (x - (-1))(x^2 - x + 1)$

$P(x) = (x + 1)(x^2 - x + 1)$

$x^2 - x + 1 = 0$

$a = 1$ $\qquad$ $b^2 - 4ac = (-1)^2 - 4(1)(1) = 1 - 4 = -3$

$b = -1$ $\qquad$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-(-1) \pm \sqrt{-3}}{2(1)} = \dfrac{1 \pm i\sqrt{3}}{2}$

$c = 1$

$\boxed{x = \frac{1}{2} \pm \frac{\sqrt{3}}{2} i}$

1 Neg.
2 Complex Zeros